**Problem 1:** A nasty bacterium in the shape of a spheroid with principle axis b, b, a and uniform density is spinning in free space about its axis of symmetry with angular velocity . The symmetry axis of the bacterium is inclined at angle with respect to an axis OP fixed in space, and precesses around it with angular velocity .

1. **Determine**

First we need to solve for the inertia tensor. The elements are computed by the following:

We will make the following coordinate transformation to a modified set of spherical coordinates to make the integration easier.

The determinate of the Jacobian will let us transform our coordinates:

Now we can rewrite the integral, it is much simpler under these new coordinates!

I computed the elements in Mathematica with the following code:

p=M/(4\*Pi\*a\*b^2/3);

r= {b\*rho\*Sin[theta]\*Cos[phi],b\*rho\*Sin[theta]\*Sin[phi],a\*rho\*Cos[theta]};

integrand[r\_]= a\*b^2\*p\*rho^2\*Sin[theta]\*(KroneckerDelta[i,j]\*(r[[1]]^2+r[[2]]^2+r[[3]]^2)-r[[i]]\*r[[j]]);

CanonicalInertia= Table[Integrate[integrand[r],

{rho, 0,1},{phi,0,2\*Pi},{theta,0, Pi}], {i, 3}, {j, 3}

];

MatrixForm[CanonicalInertia]

Where M is the mass of the ellipsoid. In the code I wrote in the volume of the ellipsoid , thus the density is .

The inertia tensor (and corresponding components of inertia) are the following:

Notice that as expected from the geometry of the bacterium.

The angular frequency is calculated from Euler’s equations for rigid bodies and setting for zero torque.

Noting that , we obtain from the third relationship:

We find that is a constant of motion in the body-fixed axis.

The first two Euler’s equations simplify to the following coupled differential equations:

If we define a positive constant ().

We can use the strategy from class: Multiply the second equation by the imaginary number and define , we get the following solution when solving for .

The solution of our new equation is:

The solution is then

If we choose the initial conditions and , the solution becomes:

Relating the imaginary and real components, we can write the entire angular velocity vector in the body-fixed axis as:

In the body-fixed axis, the angular velocity vector rotates around with frequency .

In the body-fixed axis:

If we let the be in the and plane to begin with, then is the following time-dependent vector:

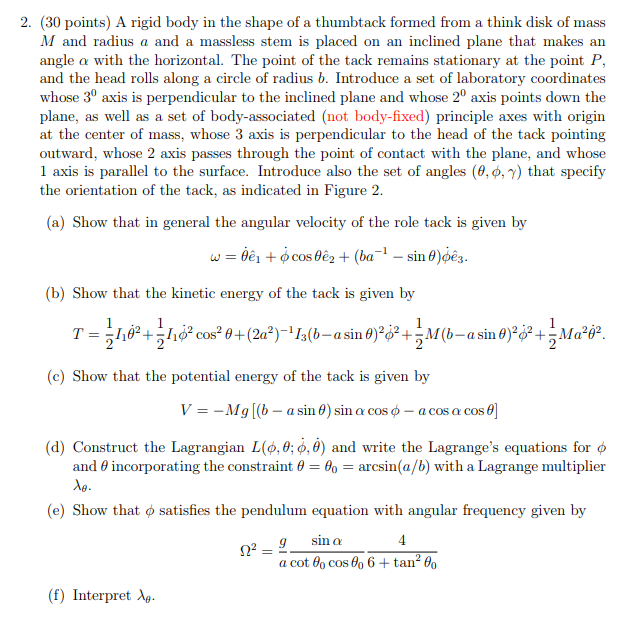
Since and make a triangle with angle :

The vector is then:

1. The nasty bacterium finds life in space difficult and transforms into a spherical spore of the same uniform density and radium c. Assume no external forces or torque have acted on the nasty bacterium, find the rotational frequency of this sphere in terms of and .

Since the bacterium turned into a sphere, we have .

The frequency becomes:



Solution:

The angular frequency can be expressed in terms of the body-associated coordinate system and inertial coordinate systems like so:

The inertial coordinate axis is expressible in terms of the body-associated coordinate axis by:

Because the line “rolled out” by the tack’s radius is equal to the line “rolled out” by the radius from the plane to the point of contact , where I have defined to be the angle between the line from the origin to the point of contact and . Since , we find that . Looking at the geometry of the tack, we also find that , so .

We rewrite now:

The inertia tensor for one of the axes is easy because it is just the inertia about a disk.

The other two side requires an integration:

The inertia tensor is then:

The rotational kinetic energy is

The translational kinetic energy is:

The total kinetic energy is then the sum:

Which can also be written:

Although I worked for several hours trying to get the potential,I could not derive the potential correctly. My attempt to find the potential became the following:

The answer should be:

If we take , then . Our Lagrangian is then:

I will rewrite the Lagrangian to make life easier: